The purpose and the definition of the CPI

Presentation by Cem BAŞ, TURKSTAT
Overview

1. Different uses of the CPI
2. CPI as a measure of inflation or cost of living?
3. Practical differences - COLI and inflation index
4. Target indices for the CPI
5. Elementary aggregates
6. Calculation of elementary price indices
7. Use of weights in elementary indices
8. Chained or direct elementary indices?
9. Conclusion
1. Different uses of the CPI

Most CPIs are used for many purposes:

- as a measure of the general rate of consumer price inflation
- as a measure of changes in the cost of living
- deflation of national accounts series
- indexation of wages, pensions and the like
- indexation of private contracts
2. CPI as a measure of inflation or cost of living

The literature distinguish between two types of consumer price indices:

- *Inflation or fixed basket* price indices
- *Cost of living* indices (COLIs)
2. CPI as a measure of inflation or cost of living

**Inflation or fixed basket index:**
- Measures the average price change of a basket of goods and services that is kept constant over time
- A fixed basket index is a *Lowe* price index:

\[
I_{0:t}^{Lo} = \frac{\sum p_t^i q_b^i}{\sum p_0^i q_b^i}
\]

- Lowe is a general type of basket index – the basket can refer to any period or combination of periods
- The index compiler needs to select the weight reference period and use expenditure shares rather than quantities
2. CPI as a measure of inflation or cost of living

**Cost of living index:**

\[
I_{0:t}^{COLI} = \frac{C(U, p_t^i)}{C(U, p_0^i)}
\]

- C(U,P) is the cost of maintaining the reference level of utility, U, in period 0 and t
- The quantities are allowed to vary in the periods compared
- Cannot be calculated in practice – needs to be approximated
3. Practical differences between COLI and inflation index

Types of acquisition

<table>
<thead>
<tr>
<th>Inflation index</th>
<th>COLI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Purchases in monetary transactions</td>
<td>May also include own-account production and social transfers in kind</td>
</tr>
<tr>
<td>Include only actual observed market prices</td>
<td>May also use imputed prices – necessary when there are no market transactions</td>
</tr>
<tr>
<td>Should be reflected in both weights and prices! Weighting data may include non-monetary consumption</td>
<td>Should be reflected in both weights and prices! Imputed prices not always easy to obtain!</td>
</tr>
</tbody>
</table>
## 3. Practical differences between COLI and inflation index

### Population coverage

<table>
<thead>
<tr>
<th>Inflation index</th>
<th>COLI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Domestic concept</td>
<td>National concept</td>
</tr>
<tr>
<td>Include consumption on domestic territory by households, also consumption by foreign households/tourists, and by institutional households</td>
<td>Include consumption by the resident population home and abroad</td>
</tr>
<tr>
<td>Consumption by foreigners difficult to measure: Usually not included in the HBS; estimates may be obtained from NA or other sources</td>
<td>Difficult to follow price development abroad! In practice consumption abroad is usually left out</td>
</tr>
</tbody>
</table>
3. Practical differences between COLI and inflation index

**Owner-occupied housing**

<table>
<thead>
<tr>
<th>Inflation index</th>
<th>COLI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Net acquisition approach</td>
<td>Rental equivalent or user cost approach</td>
</tr>
<tr>
<td>Include the actual purchase of dwellings by the households: Newly build houses, houses bought from other sectors and self-constructed houses. Excludes the use of houses!</td>
<td>Include the consumption of house service (shelter) by equivalent rents or by estimating the user costs. Excludes the acquisition of houses</td>
</tr>
<tr>
<td>Should be reflected in both weights and prices! Difficult to obtain good and timely data</td>
<td>Rental equivalent: difficult if the rental market is small or little/no connection between markets. User cost: What costs should be included?</td>
</tr>
</tbody>
</table>
3. Practical differences between COLI and inflation index

**Own account production (OAP)**

<table>
<thead>
<tr>
<th>Inflation index</th>
<th>COLI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Goods and services produced by households for their own consumption. Includes food, particularly important in rural households, services of OOH and other services, cooking, child caring, washing, cleaning etc.</td>
<td>Price changes influence the opportunity costs of household for consuming their own production. OAP of goods and OOH services included in household final consumption expenditure (SNA)</td>
</tr>
<tr>
<td>Actual market prices are not available but has to be estimated. There are no monetary transaction Imputed prices does not add new information to inflation calculation</td>
<td>Should be included. Other services (cooking, washing etc.) excluded because suitable prices cannot be found.</td>
</tr>
<tr>
<td>Should be excluded</td>
<td></td>
</tr>
</tbody>
</table>
4. Target indices for the CPI

Steps in developing the CPI

1. Consultation with main users of the CPI
2. Define the (main) purpose of the index
3. Define the scope and the actual coverage of the index
4. Select an ideal target price index
5. Decide which calculation formulas to apply in practice
4. Target indices for the CPI

What is a target index?
- An ideal index that in principle may be calculated on the basis of information of prices and quantities/expenditures

Why is a target index useful?
- It provides a reference frame for the practical compilation of the CPI
- Necessary with a measurable target to quantify the size of any potential bias:
  \[ \text{bias} = \text{target CPI} - \text{actual CPI} \]
4. Target indices for the CPI

Walsh and Marshall-Edgeworth are good fixed basket indices

\[
I_{0:t}^W = \sum p_t^i \sqrt{q_0^i \cdot q_t^i} = \sum w_W^i \left( \frac{p_t^i}{p_0^i} \right)
\]

Walsh

\[
w_W^i = \frac{\sqrt{(w_0^i \cdot w_t^i) \left( p_t^i / p_0^i \right)}}{\sum \sqrt{(w_0^i \cdot w_t^i) \left( p_t^i / p_0^i \right)}}
\]
4. Target indices for the CPI

\[ I_{0:t}^{ME} = \frac{\sum p_t^i (q_0^i + q_t^i)}{\sum p_0^i (q_0^i + q_t^i)} / 2 = \sum w_{ME}^i \left( \frac{p_t^i}{p_0^i} \right) \]

\[ w_{ME}^i = \frac{\nu_0^i + \left( \nu_t^i \left( p_t^i / p_0^i \right) \right)}{\sum \left( \nu_0^i + \left( \nu_t^i \left( p_t^i / p_0^i \right) \right) \right)} \]

\[ \nu_t^i = p_t^i q_t^i \]
4. Target indices for the CPI

Fisher and Törnqvist price indices are good COLIs:

\[
I_{0:t}^F = \left( \frac{\sum p_t^i q_0^i \sum p_t^i q_t^i}{\sum p_0^i q_0^i \sum p_0^i q_t^i} \right)^{1/2}
\]

\[
I_{0:t}^T = \prod \left( \frac{p_t^i}{p_0^i} \right)^{(w_0^i + w_t^i)/2}
\]
4. Target indices for the CPI

The CPI Manual concludes:

“Fisher, Walsh and Törnqvist price indices approximate each other very closely using “normal” time series data. This is a very convenient result since these three index number formulae repeatedly show up as being “best” in all the approaches to index number theory. Hence, this approximation result implies that it normally will not matter which of these indices is chosen as the preferred target index for a consumer price index.”

(The CPI Manual, 17.3)
5. Elementary aggregates

The typical aggregation structure

- Overall index
- Higher-level indices
- Elementary Indices
  - Individual price observations
  - Value shares (weights)
5. Elementary aggregates

Grouping of elementary aggregates:

- **Products** – goods or services – that are as similar as possible, i.e. homogeneous
- **Group products with similar price movements to minimize expected dispersion of price movements**

In the absence of weights for the individual price observations there are 3 main formulas for calculating elementary indices ...
6. Calculation of elementary price indices

**Carli index** – the arithmetic mean of the price ratios

\[
P_{0:t}^C = \frac{1}{n} \sum \left( \frac{p_t^i}{p_0^i} \right)
\]

**Dutot index** – the ratio of arithmetic mean prices

\[
P_{0:t}^D = \frac{1}{n} \sum p_t^i = \frac{1}{n} \sum \left( \frac{p_t^i}{p_0^i} \right) \cdot p_0^i
\]
6. Calculation of elementary price indices

**Jevons index** – the geometric mean of the price ratios

\[ P_{0:t}^J = \prod \left( \frac{p_t^i}{p_0^i} \right)^{1/n} = \frac{\prod (p_t^i)^{1/n}}{\prod (p_0^i)^{1/n}} \]
### 6. Calculation of elementary price indices

**Example 1: Dutot, Carli and Jevons**

<table>
<thead>
<tr>
<th></th>
<th>January</th>
<th>February</th>
<th>Feb/Jan</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>10</td>
<td>8</td>
<td>0,8</td>
</tr>
<tr>
<td>B</td>
<td>14</td>
<td>14</td>
<td>1</td>
</tr>
<tr>
<td>C</td>
<td>20</td>
<td>24</td>
<td>1,2</td>
</tr>
<tr>
<td><strong>Arithmetic mean</strong></td>
<td>14,67</td>
<td>15,33</td>
<td></td>
</tr>
<tr>
<td><strong>Geometric mean</strong></td>
<td>14,09</td>
<td>13,90</td>
<td></td>
</tr>
<tr>
<td>Dutot</td>
<td>$\frac{15,33}{14,67} \times 100$</td>
<td>=</td>
<td>104,5</td>
</tr>
<tr>
<td>Carli</td>
<td>$(0,8 + 1 + 1,2)/3 \times 100$</td>
<td>=</td>
<td>100,0</td>
</tr>
<tr>
<td>Jevons</td>
<td>$\frac{13,90}{14,09} \times 100$</td>
<td>=</td>
<td>98,6</td>
</tr>
<tr>
<td></td>
<td>$\sqrt[3]{0,8 \times 1 \times 1,2} \times 100$</td>
<td>=</td>
<td>98,6</td>
</tr>
</tbody>
</table>
### Example 2: Substitution effect in the Jevons index

<table>
<thead>
<tr>
<th></th>
<th>May</th>
<th>June</th>
<th>June/May</th>
</tr>
</thead>
<tbody>
<tr>
<td>Item A</td>
<td>10</td>
<td>12</td>
<td>1,20</td>
</tr>
<tr>
<td>Item B</td>
<td>10</td>
<td>8</td>
<td>0,80</td>
</tr>
<tr>
<td>Arithm. Mean</td>
<td>10,00</td>
<td>10,00</td>
<td>1,00</td>
</tr>
<tr>
<td>Geomean</td>
<td>10,00</td>
<td>9,80</td>
<td>0,98</td>
</tr>
<tr>
<td>Carli</td>
<td>100,0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dutot</td>
<td>100,0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jevons</td>
<td>98,00</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Carli and Dutot keeps the implicit quantities constant
Jevons allows some substitution - households consume more of B and less of A!
6. Calculation of elementary price indices

Example 3: Upward bias in Carli

<table>
<thead>
<tr>
<th></th>
<th>May</th>
<th>June</th>
<th>June/May</th>
</tr>
</thead>
<tbody>
<tr>
<td>Item A</td>
<td>20</td>
<td>25</td>
<td>1,25</td>
</tr>
<tr>
<td>Item B</td>
<td>25</td>
<td>20</td>
<td>0,80</td>
</tr>
<tr>
<td>Arithm. Mean</td>
<td>22,50</td>
<td>22,50</td>
<td>1,00</td>
</tr>
<tr>
<td>Geomean</td>
<td>22,36</td>
<td>22,36</td>
<td>1,00</td>
</tr>
</tbody>
</table>

Carli  =  \frac{(1,25 + 0,80)}{2} \times 100  =  102,5
Dutot  =  100,0
Jevons =  100,0

Carli gives more weight to price increases than to decreases!
6. Calculation of elementary price indices

Example 4: Dutot depends on the price level

<table>
<thead>
<tr>
<th></th>
<th>December</th>
<th>January</th>
<th>Index</th>
</tr>
</thead>
<tbody>
<tr>
<td>Product 1</td>
<td>25,53</td>
<td>16,06</td>
<td>62,9</td>
</tr>
<tr>
<td>Product 2</td>
<td>69,5</td>
<td>69,5</td>
<td>100,0</td>
</tr>
<tr>
<td>Product 3</td>
<td>201,67</td>
<td>221,67</td>
<td>109,9</td>
</tr>
<tr>
<td>Av. Price</td>
<td>98,9</td>
<td>102,4</td>
<td></td>
</tr>
<tr>
<td>Dutot index</td>
<td>102,4/98,9 * 100 =</td>
<td>103,5</td>
<td></td>
</tr>
<tr>
<td>Carli index</td>
<td>(62,9+100+109,9)/3 * 100 =</td>
<td>90,9</td>
<td></td>
</tr>
</tbody>
</table>

Price changes in Dutot are weighted according to the price in the reference period:

<table>
<thead>
<tr>
<th></th>
<th>Price</th>
<th>Price weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Product 1</td>
<td>25,53</td>
<td>0,09</td>
</tr>
<tr>
<td>Product 2</td>
<td>69,5</td>
<td>0,23</td>
</tr>
<tr>
<td>Product 3</td>
<td>201,67</td>
<td>0,68</td>
</tr>
<tr>
<td>Sum</td>
<td>296,7</td>
<td>1,00</td>
</tr>
<tr>
<td>Dutot index</td>
<td>62,9<em>0,09+100</em>0,23+109,9*0,68 =</td>
<td>103,5</td>
</tr>
</tbody>
</table>
6. Calculation of elementary price indices

How to decide which formula to apply?

- The *economic approach* - focuses on the economic interpretation of the index

- The *axiomatic* or *test approach* - focuses on the statistical properties of the index
6. Calculation of elementary price indices

The economic approach:

- Assume utility maximizing households with perfect information. The cost of living index is the ratio of the minimum expenditures of keeping constant utility:

\[ \text{COLI}_{0,t} = \frac{C(p_t^i, U)}{C(p_0^i, U)} \]

- The basket may change in response to consumer substitution
- Usually, quantities are not available in practice
- The assumptions are often not realistic

=> Difficult to calculated a COLI in practice
6. Calculation of elementary price indices

The axiomatic approach:
Select a number of tests – axioms – that the index should meet.

The more important tests:

**Proportionality:** If all prices change $x\%$, the index should also change by $x\%$

**Commensurability:** The index should be invariant compared to the unit in which prices are recorded

**Time reversal:** The index from period 0 to period $t$ should equal the reciprocal of the index from $t$ to 0

**Transitivity:** The index from 0 to 1 multiplied (*chained*) by an index from 1 to 2 should equal a *direct* index from 0 to 2.
6. Calculation of elementary price indices

<table>
<thead>
<tr>
<th>Proportionality</th>
<th>Carli</th>
<th>Dutot</th>
<th>Jevons</th>
</tr>
</thead>
<tbody>
<tr>
<td>Commensurability</td>
<td>yes</td>
<td>no</td>
<td>yes</td>
</tr>
<tr>
<td>Time reversal</td>
<td>no</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Transitivity</td>
<td>no</td>
<td>yes</td>
<td>yes</td>
</tr>
</tbody>
</table>

- Carli fails time reversal and transitivity
- Dutot fails commensurability
- Jevons passes all four

☞ Jevons recommended as the preferred index in general
7. Use of weights in elementary indices

Weighted averages

Laspeyres price index

\[ P_{0:t}^{La} = \frac{\sum (p_t^i q_0^i)}{\sum (p_0^i q_0^i)} = w_0 \sum \left( \frac{p_t^i}{p_0^i} \right) \]

Geometric Laspeyres price index

\[ P_{0:t}^{GLa} = \prod \left( \frac{p_t^i}{p_0^i} \right)^{w_i} = \frac{\prod (p_t^i)^{w_i}}{\prod (p_0^i)^{w_i}} \]
7. Use of weights in elementary indices

Joseph LOWE, 1823 → **FIXED BASKET** \( I = \frac{p_1 \cdot q}{p_0 \cdot q} \)

Étienne LASPEYRES, 1871 → \( q = q_0 \)

\( PL = \sum p_1 \cdot q_0 / p_0 \cdot q_0 \)

Hermann PAASCHE, 1874 → \( q = q_1 \)

\( PP = \sum p_1 \cdot q_1 / p_0 \cdot q_1 \)

Francis Ysidro EDGEWORTH
Alfred MARSHALL

Correa Mylan WALSH, 1901 → \( q = (q_0 \cdot q_1)^{1/2} \)
7. Use of weights in elementary indices

Henry SIDGWICK, 1883  \[ PS = \frac{PL + PP}{2} \]

Irving FISHER, 1922  \[ PF = (PL \cdot PP)^{1/2} \]
8. Chained or direct elementary indices?

- **A direct index** compares the prices of the current month with those of a *fixed* reference month

  \[ P_{0:t} = P_{0:t} \left( p_0, p_t \right) \]

- **A chained index** compares month-on-month price changes and multiplies the monthly indices into a long-term index

  \[ P_{0:t} = P_{0:t} \left( p_0, p_1, p_2, \ldots, p_{t-1}, p_t \right) = P_{0:1} \cdot P_{1:2} \cdot P_{2:3} \cdot \ldots \cdot P_{t-1:t} \]

- **Direct = chained index** when based on average prices, \( \overline{p}_t \), and no replacements

  \[ P_{0:t} = \frac{\overline{p}_1}{\overline{p}_0} \cdot \frac{\overline{p}_2}{\overline{p}_1} \cdot \frac{\overline{p}_3}{\overline{p}_2} \cdot \ldots \cdot \frac{\overline{p}_t}{\overline{p}_{t-1}} \cdot \frac{\overline{p}_t}{\overline{p}_0} = \overline{p}_t \]
8. Calculation of elementary price indices

Example 5: A chained Carli index is biased upwards

<table>
<thead>
<tr>
<th></th>
<th>Prices</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Jan.</td>
<td>Feb.</td>
<td>March</td>
</tr>
<tr>
<td>A</td>
<td>40</td>
<td>45</td>
<td>44</td>
</tr>
<tr>
<td>B</td>
<td>60</td>
<td>55</td>
<td>66</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Monthly price ratios</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1,13</td>
<td>0,98</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>0,92</td>
<td>1,20</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Monthly index</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>102,1</td>
<td>108,9</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Chained monthly index</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>100</td>
<td>102,1</td>
<td>111,2</td>
</tr>
</tbody>
</table>
9. Conclusions

- A clear definition of the main purpose(s) of the index is useful for the users and for the statistical office and provides guidance on geographical, population and product coverage.

- Selection of an ideal target index provides a reference frame and is needed for calculation of bias.

- Whether the purpose is to measure pure price changes or the cost of living, Fisher, Walsh and Törnqvist (*superlative* indices) are best options.

- For practical purposes the three indices can be expected to give very similar results.
9. Conclusions

- The superlative indices all use weights from the current period, which are not available in real time!

- The CPI has to be calculated on the basis of available weighting and price data.

\[ CPI^{0:t} = \sum w_i^b P_i^{0:t} \]

- Superlative indices can be calculated retrospectively for the evaluation of the CPI.
9. Conclusions

- Group homogenous products with similar expected price movements into elementary aggregates
- Carli and Jevons are independent of the price levels – Dutot depends on the initial price levels
- A chained Carli is upward biased and should not be used
- The Dutot index should only be used for homogenous elementary aggregates
9. Conclusions

- Jevons is the generally recommended index because of better statistical properties.
- Monthly chained indices appear to have some practical advantages in the treatment of missing prices and replacements.
- Explicit weights may be applied for the calculation of elementary indices.
- Without explicit weights, there will still be implicit weighting from the sampling!
Thank you...