"Calculation of higher-level indices"

20-22 September 2021

TURKSTAT

The compilation of higher-level indices

This section covers the compilation of upper-level aggregation, sometimes known as macro-indices.

Upper-level aggregate indices constructed as weighted averages of elementary aggregate indices.

The compilation of higher-level indices

Choice of index formula.

- In principle, the choice of index formula used to calculate a CPI is determined by the "target" index.
  - Cost-of-living or pure price index.
  - Walsh, Edgeworth-Marshall or Törnqvist price indices.
- Available weights information is a constraint.
- What follows focuses on macro level indices using the class of fixed-basket formula most statistical agencies use to construct CPIs.

The compilation of higher-level indices

Laspeyres or Laspeyres-type index.

A true Laspeyres - the weight reference period must coincide with time *b.*

$$P_{LA}^{t/b} = \frac{\sum_i p_i^t q_i^b}{\sum_i p_i^b q_i^b}$$

where:

- *i is the number of products in the index basket.*
- *p_i^b is the price of the i th product at period b, the base or reference period.*
- *p_i^t is the price of the i th product at period t (t > 0).*
- *q_i^b is the quantity of the i th product at period b, the base or reference period.*

Principles of price collections

A “true” Laspeyres price index, where the base period coincides with time b, is rarely a practical option for a timely CPI.

- For seasonal products the expenditure at time *b*, a sub-annual period, may be unrepresentative of expenditures at other periods.
- Expenditure data for periods less than a full year are often unreliable.
- The reference period for the expenditure data are not in sync with the chosen basket update period.

The compilation of higher-level indices

- Price updating of expenditure weights provides a solution to bringing weight and price reference periods into sync.
- Once the weights are price-updated, the resulting price index is more in line with what is commonly recognised as a Lowe index (or Laspeyres-type price index).

$$P_{Lo}^{t/0} = \frac{\sum_i p_i^t q^b}{\sum_i p_i^0 q^b}$$

The compilation of higher-level indices

- An equivalent algebraic transformation of formula is:

$$P_{LO}^{t/b} = \sum_i \frac{p_i^t}{p_i^0} \frac{p_i^0 q_i^b}{\sum_i p_i^0 q_i^b}$$

- The following expression is the basket share of product (or elementary aggregate) *i* *(expressed in parts per 100).*

$$\frac{p_i^0 q_i^b}{\sum_i p_i^0 q_i^b}$$

The compilation of higher-level indices

- The index is interpreted as the arithmetic average of the price relatives between two periods for a given set of goods and services covered by the index.

$$P_{LO}^{t/0} = \sum_i \frac{p_i^t}{p_i^0} \frac{w_i}{\sum_i w_i}$$

The prices to be collected

Expressing the price index as a weighted average-of-relatives does have other advantages over the aggregative indices:

- The price relatives for each individual product in the aggregate, together constitute a simple product price index that provides analytical information.
- When a new commodity is introduced to replace a former one. the relative for the new item may be spliced to the relative for the old one.

The compilation of higher-level indices

The weights of the individual products that are used in the previous formula (w_i) are defined as "hybrid" weights that have been price-updated from the basket reference period.

The ps in the formula are more accurately be described as an estimate of the price of a group or sample of products that make up the elementary index aggregate.

The compilation of higher-level indices

Time-lags and the use of price-updated weights in a Laspeyres-type index.

- As previously mentioned, the price reference and weight reference periods coincide in the formulation of the true Laspeyres price index.
- In practice the price base period and the basket reference period will diverge because of an unavoidable lag associated with weights data.
- Weights commonly used in a CPI are not the observed weights from their reference period but can best be described as hybrid.

Be careful-----

$$P_{LA}^{t/b} = \sum_i \frac{p_i^t}{p_i^b} \frac{p_i^b q_i^b}{\sum_i p_i^b q_i^b}$$

The compilation of higher-level indices

Multiplying the reference period expenditure for that commodity by its corresponding elementary index for period *b* to period *0*, will yield that commodity's price-updated hybrid weight.

Given the set of price-updated weights, the formula for calculating the CPI from period 0 to period *t* can then be re-written.

The compilation of higher-level indices

The chain link method is operationally more convenient.

Hybrid weights are present. Most countries do not use the following equation to calculate a CPI.

$$P_{LO}^{t/0} = \sum_i \frac{p_i^t}{p_i^0} \frac{p_i^0 q_i^b}{\sum_i p_i^0 q_i^b}$$

The compilation of higher-level indices

A more operationally convenient variant is often used instead.

This is the "procedural" monthly *chain-link* method, which can be calculated for period t as follows:

$$P_{LO}^{t/0} = \sum_i \frac{p_i^t q_i^b}{p_i^0 q_i^b} = \sum_i \frac{p_i^t}{p_i^{t-1}} \frac{p_i^{t-1} q_i^b}{\sum_i p_i^0 q_i^b}$$

The compilation of higher-level indices

Calculating the index using the monthly chain-link formula has two significant advantages from an operational point of view:

- The short term relative procedure offers more latitude to the compiler for dealing with new and disappearing products, missing prices, and quality adjustments.
- Any month can potentially be used as a link period.

Thank you…